A critical synthesis of thermophysical characteristics of nanofluids

Khalil Khanafer, Kambiz Vafai

1. Introduction

Recent advances in nanotechnology have led to the development of a new, innovative class of heat transfer fluids (nanofluids) created by dispersing nanoparticles (10–50 nm) in traditional heat transfer fluids [1]. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas such as industrial cooling applications, nuclear reactors, transportation industry (automobiles, trucks, and airplanes), micro-electromechanical systems (MEMS), electronics and instrumentation, and biomedical applications (nano-drug delivery, cancer therapeutics, cryopreservation) [2]. Possible improved thermal conductivity translates into higher energy efficiency, better performance, and lower operating costs.

A large number of research work related to the heat transfer enhancement using nanofluids both experimentally and theoretically was conducted by a number of investigators [1–15]. Although various studies have shown that nanofluids illustrate higher heat transfer enhancement than those of base fluids, contradictory results on nanofluids behavior were also reported. For example, Pak and Cho [16] demonstrated that the Nusselt number for Al₂O₃–water and TiO₂–water nanofluids increased with increasing volume fraction as well as the Reynolds number. However, the convective heat transfer coefficient for nanofluids at a volume fraction of 3% was found to be 12% smaller than that of pure water when considering a constant average velocity [16]. Yang et al. [17] reported a similar conclusion for graphite nanofluids in the laminar flow regime. On the contrary, many other researchers have reported heat transfer enhancement using nanofluids [6,11,15,18–22]. For instance, Xuan and Li [15] conducted an experimental study to investigate convective heat transfer and flow features of nanofluids. Their results show that convective heat transfer coefficient of the nanofluid increased with the flow velocity as well as the volume fraction of nanoparticles and it was larger than that of the base water under the same flow velocity. Das et al. [22] had shown experimentally that the thermal conductivity of nanofluid increases with an increase in temperature. They observed that a 2 to 4-fold increase in the thermal conductivity can be achieved over the temperature range of 21–52°C.

In contrast to research investigations related to forced convective heat transfer using nanofluids, few studies are found in the literature on the use of nanofluids in natural convective heat transfer. Khanafer et al. [6] conducted a numerical study to determine natural convection heat transfer of nanofluids in an enclosure under various physical constraints. Their results illustrated that the average Nusselt number increases with an increase in the particle volume fraction for different Grashof numbers. Kim et al. [23] proposed a factor which described the effect of nanoparticle addition on the convective instability and heat transfer characteristics of a base fluid. The new factor included the effect of the ratio of the thermal conductivity of nanoparticles to that of the base fluid,
et al. [30] and Eastman et al. [31] proposed four main mechanisms for thermal conductivity enhancement of nanofluids. These include Brownian motion of nanoparticles, molecular-level layering of the liquid at the liquid/particle interface, heat transport within the nanoparticles and the effects of nanoparticle clustering. They had reasoned that the effect of Brownian motion can be neglected due to the greater input of the thermal diffusion as compared to the Brownian diffusion. Evans et al. [32] had also demonstrated that the hydrodynamics effects associated with Brownian motion have a minor effect on the thermal conductivity of the nanofluid using the molecular dynamics simulations and the simple kinetic theory.

The effect of the solid/liquid interfacial layer on the thermal conductivity enhancement of nanofluids was studied theoretically by many researchers [33–38]. For example, Yu and Choi [33,34] proposed a theoretical model for the effect of a solid/liquid interface based on the Hamilton–Crosser model for suspensions of nonspherical particles. They attempt to demonstrate that the solid/liquid interfacial layers play an important role in augmenting the thermal conductivity of nanofluids through the use of their Hamilton–Cresser model. However, their proposed model was unable to predict the nonlinear behavior of the nanofluid thermal conductivity. Considering the interface effect between the nanoparticles and base fluid, Xue [36] developed a theoretical model for the effective thermal conductivity for nanofluids based on Maxwell theory and average polarization theory. Xue [36] postulated that the developed model can interpret the anomalous enhancement of the effective thermal conductivity of the nanofluid. Other theoretical studies in the literature indicate additional conflicting results. Based on molecular dynamic simulations and simple liquid–solid interfaces, Xue et al. [38] illustrated that the layering of the liquid atoms at the liquid–solid interface does not have any considerable effect on thermal transport properties.

One can notice from the earlier cited information that neither Brownian motion nor interfacial liquid layering can be a dominant mechanism and that the findings from experimental and analytical investigations on the heat transfer enhancement using nanofluids are in disagreement for both natural and forced convection. Hence, further theoretical and experimental research are essential in order to explain the basis for possible heat transfer enhancement when using nanofluids. Although many possible mechanisms were proposed in the literature, there is no robust description of the anomalous behavior of nanofluids including higher thermal conductivity and viscosity. The thermal conductivity and viscosity data of nanofluids are still contradictory in various research publications. As such, it is still unclear as to what are the best models to use for the thermal conductivity and viscosity of nanofluids. Therefore, the aim of this study is to analyze the variants within the thermophysical characteristics of nanofluids especially with respect to the

\[ c_{\text{eff}} \] heat capacity of nanofluids  
\[ c_f \] heat capacity of the base fluid  
\[ c_p \] heat capacity of nanoparticles  
\[ d_p \] nanoparticles diameter  
\[ h \] inter-particle spacing  
\[ k \] thermal conductivity  
\[ k_B \] Stefan–Boltzmann constant  
\[ k_{l\text{ayer}} \] thermal conductivity of the nano-layer  
\[ m \] mass  
\[ n \] empirical shape factor  
\[ Pr \] Prandtl number  
\[ Re \] Reynolds number  
\[ t \] thickness of the nano-layer  
\[ T \] temperature  
\[ V \] volume

Greek symbols  
\[ \beta_{\text{eff}} \] thermal expansion coefficient of nanofluids  
\[ \beta_f \] thermal expansion coefficient of the base fluid  
\[ \beta_p \] thermal expansion coefficient of nanoparticles  
\[ \rho \] density  
\[ \rho_b \] density of nanofluids  
\[ \Psi \] particle sphericity  
\[ \phi \] volume fraction of nanoparticles  
\[ \phi_{\text{max}} \] maximum volume fraction of nanoparticles  
\[ \mu_{\text{eff}} \] dynamic viscosity of nanofluids  
\[ \mu_b \] dynamic viscosity of the base fluid  
\[ \mu_{\text{Brownian}} \] dynamic viscosity due to Brownian motion

Subscripts  
\[ f \] fluid  
\[ p \] nanoparticle

shape factor of the nanoparticles, volume fraction of nanoparticles, and the heat capacity ratio. Their results indicate that the heat transfer coefficients in the presence of nanofluids increases with an increase in the volume fraction of nanoparticles. Convective heat transfer enhancement using nanofluids was also observed experimentally by Nanna et al. [24] and Nanna and Routhu [25].

On the other hand, Putra et al. [13] found experimentally that the presence of nanoparticles (Al₂O₃, and CuO) in water based nanofluids inside a horizontal cylinder decreased natural convective heat transfer coefficient with an increase in the volume fraction of nanoparticles, particle density as well as the aspect ratio of the cylinder. Ding et al. [26] have also reported experimentally that natural convective heat transfer coefficient decreases systematically with an increase in nanoparticle concentration, and the deterioration was partially attributed to the higher viscosity of nanofluids. Chang et al. [27] performed natural convection experiments with Al₂O₃ microparticle (~250 nm) aqueous suspensions in thin enclosures. Their results seem to indicate that the particles have a negligible effect on the Nusselt number values for a vertical enclosure. However, for horizontal enclosure, there was a decrease in Nusselt number compared to presence of pure water at lower Rayleigh numbers and higher particle concentrations. The authors attributed this anomalous behavior to sedimentation.

Ho et al. [28] had experimentally investigated natural convection heat transfer of a nanofluid in a vertical enclosure for different particle sizes and various volume fractions of nanoparticles (Al₂O₃) ranging from 0.1% to 4% and Rayleigh number variations in the range of \( 10^5 \)–\( 10^8 \). Systematic heat transfer degradation was observed in their measurements for nanofluids containing nanoparticles with volume fractions greater than 2% over the entire range of Rayleigh numbers. However, heat transfer enhancement around 18% compared with pure water was exhibited for nanofluid containing lower nanoparticle concentrations of 0.1% at high Rayleigh numbers.

Currently, there are no reliable theoretical models to determine the anomalous thermal conductivity of nanofluids. Many researchers have based the thermal conductivity of nanofluids on thermal conductivities of fluid and nanoparticles, shape and surface area of nanoparticles, volume fraction, and temperature [29]. Kebinski et al. [30] and Eastman et al. [31] proposed four main mechanisms for thermal conductivity enhancement of nanofluids. These include Brownian motion of nanoparticles, molecular-level layering of the liquid at the liquid/particle interface, heat transport within the nanoparticles and the effects of nanoparticle clustering. They had
thermal conductivity and viscosity models and propose possible physical reasons for the deviations between experimental and analytical studies.

1.1. Analytical models for physical properties of nanofluids

1.1.1. Density

The density of nanofluid is based on the physical principle of the mixture rule. As such it can be represented as:

$$\rho_{\text{eff}} = \frac{m}{V_{\text{eff}}} = \frac{m_f + m_p}{V_f + V_p} = \frac{\rho_f V_f + \rho_p V_p}{V_f + V_p} = (1 - \phi_p)\rho_f + \phi_p\rho_p,$$

where $f$ and $p$ refer to the fluid and nanoparticle respectively and $\phi_p = \frac{V_p}{V_f + V_p}$ is the volume fraction of the nanoparticles. To examine the validity of Eq. (1), Pak and Cho [16] and Ho et al. [28] conducted experimental studies to measure the density of Al$_2$O$_3$–water nanofluids at room temperature as depicted in Fig. 1a. Fig. 1a shows an excellent agreement between the experimental results and the predictions using Eq. (1).

The physical properties at different temperatures are very important for engineering calculations. Ho et al. [28] measured the density of Al$_2$O$_3$–water nanofluid at different temperatures and nanoparticle volume fractions. In the present work, a correlation for the density of Al$_2$O$_3$–water nanofluid using the experimental data of Ho et al. [28] as a function of temperature and volume fraction of nanoparticles has been developed. The present developed correlation can be expressed as:

$$\rho_{\text{eff}} = 1001.064 + 2738.6191\phi_p - 0.2095T; \quad 0 \leq \phi_p \leq 0.04, \quad 5 \leq T(\degree C) \leq 40.$$

The $R^2$ of the regression is 99.97% and the maximum relative error is 0.22%. It is clear from Fig. 1b that the current regression (Eq. (2)) is in excellent agreement with the measurements of Ho et al. [28]. In addition, Fig. 1b shows that the rate of decrease of the effective density of Al$_2$O$_3$–water nanofluid with increasing temper-

![Fig. 1. Effect of the volume fraction on the density of the Al$_2$O$_3$–water nanofluid (a) room temperature; (b) various temperatures.](image-url)
ature is insignificant. This is due to the fact that the density of the Al₂O₃ nanoparticles is even less sensitive to the temperature when compared to the density of water.

2. Heat capacity of nanofluids

The specific heat of nanofluid can be determined by assuming thermal equilibrium between the nanoparticles and the base fluid phase as follows:

\[
(\rho c)_{\text{eff}} = \rho_{\text{eff}} \left( \frac{Q}{m_\Delta T} \right)_{\text{eff}} = \rho_{\text{eff}} \left( \frac{Q_f + Q_p}{(m_f + m_p)\Delta T} \right)
\]

where

\[
Q = (mc)\Delta T + (mc)_f\Delta T = (mc)_{\text{eff}}\Delta T
\]

\[
\rho_{\text{eff}} = \frac{\rho_f V_f + \rho_p V_p}{\rho_f V_f + \rho_p V_p}
\]

\[
(c)_{\text{eff}} = \frac{(1 - \phi_p)c_f + \phi_p(c_f + c_p)}{\rho_{\text{eff}}}
\]

(3)

where \(\rho_p\) is the density of the nanoparticle, \(\rho_f\) is the density of the base fluid, \(\rho_{\text{eff}}\) is the density of the nanofluid, and \(c_f\) and \(c_p\) are the heat capacities of the nanoparticle and the base fluid, respectively. However, some authors [16,22,39–41] prefer to use a simpler expression given as:

\[
c_{\text{eff}} = (1 - \phi_p)c_f + \phi_p c_p
\]

(4)

Fig. 2 shows a comparison of the specific heat of Al₂O₃–water nanofluid at room temperature using both equations with the experimental data of Zhou and Ni [42] for various volume fractions (\(\phi_p = 0–21.7\%\)). This figure shows that the specific heat of the nanofluid based on the models given in Eqs. (3) and (4) decreases with an increase in the volume fraction of nanoparticles. The experimental results were compared with the predictions obtained from the models given in Eqs. (3) and (4) as shown in Fig. 2. Fig. 2 shows that model I compares very well with the experimental data of Zhou and Ni [42].

3. Thermal expansion coefficient of nanofluids

The thermal expansion coefficient of nanofluids can be estimated utilizing the volume fraction of the nanoparticles on a weight basis as follows [6]:

\[
\beta_{\text{eff}} = \frac{(1 - \phi_p)\beta_f + \phi_p(\beta_f + \beta_p)}{\rho_{\text{eff}}},
\]

(4)

where \(\beta_f\) and \(\beta_p\) are the thermal expansion coefficients of the base fluid and the nanoparticle, respectively. A simpler model for the thermal expansion coefficient of the nanofluid is suggested as [43,44]:

\[
\beta_{\text{eff}} = (1 - \phi_p)\beta_f + \phi_p \beta_p.
\]

(5)

To the best of our knowledge, there is one experimental measurement of the volumetric expansion coefficient of the nanofluid that has been reported in the literature [28]. Ho et al. [28] conducted an experimental study to determine the thermal expansion of Al₂O₃–water nanofluid at various volume fractions of nanoparticles. For water, \(\beta_f\) varies from \(1.5 \times 10^{-4} \text{°C}^{-1}\) to \(6.2 \times 10^{-4} \text{°C}^{-1}\) over a temperature range of \(15 \text{°C} \leq T \leq 80 \text{°C}\), which is two orders of magnitude higher than \(\beta_p\). The values of the thermal expansion of Al₂O₃–water nanofluid predicted by Eqs. (4) and (5) were compared with the experimental data of Ho et al. [28] at a temperature of 26 °C. Fig. 3a shows that neither Eq. (4) nor Eq. (5) can be used to properly estimate the thermal expansion of nanofluid as compared to the experimental data of Ho et al. [28].

Ho et al. [28] also studied the effect of temperature and volume fraction of nanoparticles on the thermal expansion coefficient of Al₂O₃–water nanofluid. A correlation for the thermal expansion coefficient of Al₂O₃–water nanofluid as a function of temperature and volume fraction of nanoparticles based on the data presented in Ho et al. [28] has been developed in the current work. This correlation can be presented as:

\[
\beta_{\text{eff}} = \left(-0.479\phi_p + 9.3158 \times 10^{-3}T - \frac{4.7211}{T^2}\right) \times 10^{-3};
\]

\[
0 \leq \phi_p \leq 0.04, \quad 10 \text{°C} \leq T \leq 40 \text{°C}
\]

(6)

The \(R^2\) of the above correlation is 99%. The validity of this correlation [Eq. (6)] is depicted in Fig. 3b.

4. Effective viscosity of nanofluids

4.1. Analytical studies

Different models of viscosity have been used by researchers to model the effective viscosity of nanofluid as a function of volume fraction. Einstein [45] determined the effective viscosity of a suspension of spherical solids as a function of volume fraction (volume concentration lower than 5%) using the phenomenological hydrodynamic equations. This equation was expressed by:
Since Einstein’s analysis of the viscosity of a dilute suspension of rigid spheres in a viscous liquid, several equations have been developed in an effort to extend Einstein’s formula to suspensions of higher concentrations, including the effect of non-spherical particle concentrations [46–50]. For example, Brinkman [46] presented a viscosity correlation that extended Einstein’s equation to concentrated suspensions:

\[
\mu_{\text{eff}} = (1 + 2.5\phi_p)\mu_f.
\]

Since Einstein’s analysis of the viscosity of a dilute suspension of rigid spheres in a viscous liquid, several equations have been developed in an effort to extend Einstein’s formula to suspensions of higher concentrations, including the effect of non-spherical particle concentrations [46–50]. For example, Brinkman [46] presented a viscosity correlation that extended Einstein’s equation to concentrated suspensions:

\[
\mu_{\text{eff}} = \frac{1 - 2.5\phi_p}{1 - \phi_p} = (1 + 2.5\phi_p + 4.375\phi_p^2 + \cdots)\mu_f.
\]

The effect of Brownian motion on the effective viscosity in a suspension of rigid spherical particles was studied by Batchelor [47]. For isotropic structure of suspension, the effective viscosity was given by:

\[
\mu_{\text{eff}} = \left(1 + 2.5\phi_p + 6.2\phi_p^2\right)\mu_f.
\]

Lundgren [48] proposed the following equation under the form of a Taylor series in \(\phi_p\):

\[
\mu_{\text{eff}} = \frac{1}{1 - 2.5\phi_p}\mu_f = \left(1 + 2.5\phi_p + 6.25\phi_p^2 + O(\phi_p^3)\right)\mu_f.
\]

It is obvious that if the terms \(O(\phi_p^2)\) and higher are neglected, the above correlation reduces to that of Einstein. Table 1 summarizes the most common analytical expressions for the viscosity of nanofluids as a function of the volume fraction of the nanoparticles.

4.2. Experimental studies

Compared with the experimental studies on thermal conductivity of nanofluids, there are limited rheological studies reported in the literature [16,55–63]. Models of the effective viscosity of nanofluids based on the experimental data are limited to certain nanofluids. Masuda et al. [56] were the first to measure the viscosity of several water-based nanofluids for temperatures ranging from room condition to 67°C. Wang et al. [55] obtained, using three different preparation methods, some data for the dynamic viscosity of \(\text{Al}_2\text{O}_3\)-water and \(\text{Al}_2\text{O}_3\)-ethylene glycol mixtures at various temperatures.
two-phase mixture, is to some extent sufficient to estimate the viscosity of Al₂O₃–water nanofluids. For example, Nguyen et al. [60] investigated experimentally the influence of temperature on the dynamic viscosities of two particular water-based nanofluids, namely Al₂O₃–water (dₚ = 47 nm, 36 nm) and CuO–water (dₚ = 29 nm) mixtures. The following formulas were proposed by Nguyen et al. [60] for computing the dynamic viscosity for all three nanofluids tested and particle concentrations of 1% and 4%, respectively:

\[
\mu_{\text{eff}} = (1.125 - 0.0007 \times T) \mu_{T}; \quad \phi_p = 1\% \tag{11}
\]

\[
\mu_{\text{eff}} = (2.1275 - 0.0215 \times T + 0.0002 \times T^2) \mu_{T}; \quad \phi_p = 4\% \tag{12}
\]

As can be seen from Eqs. (11) and (12), Nguyen et al. [60] did not explicitly express the dynamic viscosity as a function of temperature and volume fraction. Palm et al. [68] proposed equations for the dynamic viscosity (Pa s) by means of the polynomial curve fitting based on the data reported by Putra et al. [13]. The resulting equations as a function of temperature, expressed in Kelvin, are:

For Al₂O₃–water:

\[
\mu_{\text{eff}} = 0.034 - 2 \times 10^{-4}T + 2.9 \times 10^{-7}T^2; \quad \phi_p = 1\%; \tag{13}
\]

\[
\mu_{\text{eff}} = 0.039 - 2.3 \times 10^{-4}T + 3.4 \times 10^{-7}T^2; \quad \phi_p = 4\%. \tag{14}
\]

Because the formulas such as the one proposed by Einstein [45] and later improved by Brinkman [46] and Batchelor [47] underestimate the viscosity of the nanofluids when compared to the measured data, Maiga et al. [57,58] performed a least-square curve fitting of some experimental data of Wang et al. [55] including Al₂O₃ in water and Al₂O₃ in ethylene glycol. Table 2 illustrates a summary of the viscosity models at room temperature based on the experimental data available in the literature. Moreover, Fig. 4 shows a comparison of the relative dynamic viscosity of Al₂O₃–water nanofluid from various sources at room temperature. This figure shows that Brinkman model [46], which was derived for two-phase mixture, is to some extent sufficient to estimate the viscosity for relatively low volume fraction of particles (i.e., \( \phi_p \leq 2\% \)). However, it considerably underestimates the nanofluid viscosity when compared to experimental data at high particle concentrations. The differences in the relative viscosity among the experimental data as shown in Fig. 4 may be due to the difference in the size of the particle clusters, dispersion techniques, and the methods of measurements. This clearly shows the disagreement between the researchers in measuring the viscosity of nanofluids.

4.3. Effect of temperature on the dynamic viscosity of nanofluids

It should be noted that all the above mentioned correlations were developed to relate viscosity as a function of volume fraction only; without any temperature dependence considerations. It should be mentioned that there exists a few studies in the literature associated with the effect of temperature on the viscosity of nanofluids. For example, Nguyen et al. [60] investigated experimentally the influence of the temperature on the dynamic viscosities of two particular water-based nanofluids, namely Al₂O₃–water and CuO–water mixtures. The following formulas were proposed by Nguyen et al. [60] for computing the dynamic viscosity for all three nanofluids tested and particle concentrations of 1% and 4%, respectively:

\[\mu_{\text{eff}} = (1.125 - 0.0007 \times T) \mu_{T}; \quad \phi_p = 1\% \tag{11}\]

\[\mu_{\text{eff}} = (2.1275 - 0.0215 \times T + 0.0002 \times T^2) \mu_{T}; \quad \phi_p = 4\% \tag{12}\]
Table 2
Summary of viscosity models at room temperature based on experimental data.

<table>
<thead>
<tr>
<th>Models</th>
<th>Effective viscosity (regression)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maiga et al. [57]</td>
<td>$\mu_{ef} = (1 + 7.3\phi_p + 123\phi_p^2)\mu_f$</td>
<td>- Least-square curve fitting of Wang et al. [55] data</td>
</tr>
<tr>
<td>Maiga et al. [57]</td>
<td>$\mu_{ef} = (1 - 0.19\phi_p + 306\phi_p^2)\mu_f$</td>
<td>- Al$_2$O$_3$–water, $d_p = 28$ nm</td>
</tr>
<tr>
<td>Present work</td>
<td>$\mu_{ef} = (1 + 0.164\phi_p + 302.34\phi_p^2)\mu_f$</td>
<td>- Least-square curve fitting of experimental data [55,56]</td>
</tr>
<tr>
<td></td>
<td>$d_p = 28$ nm</td>
<td>- Al$_2$O$_3$–ethylene glycol</td>
</tr>
<tr>
<td>Buongiorno [67]</td>
<td>$\mu_{ef} = (1 + 39.11\phi_p + 513.9\phi_p^2)\mu_f$</td>
<td>- Least-square curve fitting of experimental data [55,56]</td>
</tr>
<tr>
<td></td>
<td>$d_p = 28$ nm</td>
<td>- TiO$_2$–water, $d_p = 27$ nm</td>
</tr>
<tr>
<td>Present work</td>
<td>$\mu_{ef} = (1 + 23.09\phi_p + 1525.3\phi_p^2)\mu_f$</td>
<td>- Curve fitting of Pak and Cho [16] data</td>
</tr>
<tr>
<td></td>
<td>$0 \leq \phi_p \leq 0.04$</td>
<td>- CuO–water, $d_p = 29$ nm</td>
</tr>
<tr>
<td>Present work</td>
<td>$\mu_{ef} = (1 + 3.544\phi_p + 169.46\phi_p^2)\mu_f$</td>
<td>- Curve fitting of the experimental data</td>
</tr>
<tr>
<td></td>
<td>$0 \leq \phi_p \leq 0.1$</td>
<td>- TiO$_2$–water</td>
</tr>
<tr>
<td>Nguyen et al. [60]</td>
<td>$\mu_{ef} = \mu_f \times 0.904e^{0.118\phi_p}$</td>
<td>- Curve fitting of the experimental data</td>
</tr>
<tr>
<td></td>
<td>$d_p = 47$ nm</td>
<td>- Al$_2$O$_3$water</td>
</tr>
<tr>
<td>Nguyen et al. [60]</td>
<td>$\mu_{ef} = (1 + 0.025\phi_p + 0.015\phi_p^2)\mu_f$</td>
<td>- Curve fitting of the experimental data</td>
</tr>
<tr>
<td></td>
<td>$d_p = 36$ nm</td>
<td>- CuO–water, $d_p = 29$ nm</td>
</tr>
<tr>
<td>Tseng and Lin [61]</td>
<td>$\mu_{ef} = 13.47 \exp(35.98\phi_p)\mu_f$; $0.05 \leq \phi_p \leq 0.12$</td>
<td>- TiO$_2$–water</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shear rate = 100 $s^{-1}$</td>
</tr>
</tbody>
</table>

Fig. 4. Relative viscosity measurement as a function of the volume fraction, $\phi_p$, at ambient temperature. (See above-mentioned references for further information.)

Tables 3 and 4 provide a summary of different models of the dynamic viscosity of nanofluids as a function of temperature and volume fraction of nanoparticles. In the present work, a general correlation (Eq. (15)) for the effective viscosity of Al$_2$O$_3$–water, one of the most commonly studied nanofluids, is developed using various experimental data found in the literature (Fig. 5a) as a function of volume fraction, nanoparticles diameter, and temperature as follows:

$$\mu_{ef} = -0.4491 + \frac{28.837}{T} + 0.574\phi_p - 0.1634\phi_p^2 + 23.052\frac{\phi_p^3}{T^2} + 0.0132\phi_p^4 - 2354.735\frac{\phi_p}{T^2} + 23.498\frac{d_p^2}{\phi_p} - 3.0185\frac{d_p^4}{\phi_p}$$

$1\% \leq \phi_p \leq 9\%$, $20 \leq T(\degree C) \leq 70$. $13$ nm $\leq d_p \leq 131$ nm.

(15)

The $R^2$ of the regression is 99%. The validity of the above correlation (Eq. (15)) is depicted in Fig. 5b. As can be seen in Fig. 5a, the viscosity of the nanofluid decreases with an increase in the temperature. Moreover, there is no agreement between researchers about the experimentally observed magnitude of the nanofluid’s viscosity. Published results indicate a surprising range of variation of the results.

5. Thermal conductivity of nanofluids

A wide range of experimental and theoretical studies were conducted in the literature to model thermal conductivity of nanofluids. Published results illustrated neither agreement about the mechanisms for heat transfer enhancement nor a unified possible explanation regarding the rather large discrepancies in the results even for the same base fluid and nanoparticles size. Currently,
Table 3

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model (regression)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>( \mu_{eff} = 0.444 - 0.254d_p + 0.0368d_p^2 + 26.333d_p^3 - 59.311d_p^4 )</td>
<td>Curve fitting of Pak and Cho [16] data ( d_p = 13 \text{ nm} )</td>
</tr>
<tr>
<td></td>
<td>( 20 \leq T(&lt;60) \leq 70, d_p = 1.34 % - 2.78 % )</td>
<td>Units: mPa s</td>
</tr>
<tr>
<td>Palm et al. [68]</td>
<td>( \mu_{eff} = 0.034 - 2 \times 10^{-4}T(K) + 2.9 \times 10^{-6}T^2(K), \phi_p = 1 % )</td>
<td>Curve fitting of the experimental data Putra et al. [13] ( d_p = 131.2 \text{ nm} )</td>
</tr>
<tr>
<td></td>
<td>( \mu_{eff} = 0.039 - 2 \times 10^{-4}T(K) + 3 \times 10^{-6}T^2(K), \phi_p = 4 % )</td>
<td>Units: Pa s</td>
</tr>
<tr>
<td>Nguyen et al. [60]</td>
<td>( \mu_{eff} = (1.125 - 0.0007 \times T(K))\mu_I, \phi_p = 1 % )</td>
<td>Units: mPa s</td>
</tr>
<tr>
<td></td>
<td>( \mu_{eff} = (1.1275 - 0.0215 \times T(K) + 0.0002 \times T^2(K))\mu_I, \phi_p = 4 % )</td>
<td>Units: mPa s</td>
</tr>
<tr>
<td>Present work</td>
<td>( \mu_{eff} = -0.4892 + 15.9006 \phi_p + 0.6837 \phi_p + 24.1418 \phi_p^2 - 0.1785 \phi_p^3 )</td>
<td>Curve fitting of Nguyen et al. [60] data ( d_p = 47 \text{ nm} )</td>
</tr>
<tr>
<td></td>
<td>( 0.1818 \phi_p^2 + 27.015 \phi_p + 0.0132 \phi_p^2 - 2940.1775 \phi_p )</td>
<td>Units: mPa s</td>
</tr>
<tr>
<td></td>
<td>( 1 % \leq \phi_p \leq 9.4 %, 20 \leq T(K) \leq 70 )</td>
<td></td>
</tr>
<tr>
<td>Present work</td>
<td>( \mu_{eff} = -0.1011 + 28.8413 \phi_p + 0.3619 \phi_p + 144.8418 \phi_p^2 - 0.0966 \phi_p^3 )</td>
<td>Curve fitting of various experimental data available in the literature [13,60,65]</td>
</tr>
<tr>
<td></td>
<td>( 0.1609 \phi_p^2 + 22.4901 \phi_p + 0.0078809 \phi_p^2 - 2316.3754 \phi_p )</td>
<td>Units: mPa s</td>
</tr>
<tr>
<td></td>
<td>( 1 % \leq \phi_p \leq 9 %, 20 \leq T(K) \leq 70 )</td>
<td></td>
</tr>
<tr>
<td>Present work</td>
<td>( \mu_{eff} = -0.4491 + 48.8112 \phi_p + 0.5748 \phi_p - 0.1634 \phi_p^2 + 23.053 \phi_p^3 )</td>
<td>Curve fitting of various experimental data available in the literature [13,60,65]</td>
</tr>
<tr>
<td></td>
<td>( 0.0132 \phi_p^2 - 2354.735 \phi_p + 23.499 \phi_p^3 - 3.0185 \phi_p^4 )</td>
<td>Units: mPa s</td>
</tr>
<tr>
<td></td>
<td>( 1 % \leq \phi_p \leq 9 %, 20 \leq T(K) \leq 70 )</td>
<td></td>
</tr>
<tr>
<td>Namburu et al. [70,71]</td>
<td>( \log(\mu_{eff}) = Ae^{-B\phi_p} \times \text{nm Pa s} )</td>
<td>Experimental Al2O3–ethylene glycol and water mixture</td>
</tr>
<tr>
<td></td>
<td>( \phi_p = 1 % \leq \phi_p \leq 10 % d_p = 33 \text{ nm} )</td>
<td>( 238 &lt; T &lt; 323 \text{ K} )</td>
</tr>
</tbody>
</table>

There are no theoretical results available in the literature that predicts accurately the thermal conductivity of nanofluids. The existing results were generally based on the definition of the effective thermal conductivity of a two-component mixture as follows [73]:

\[
k_{eff} = \frac{k_f(1 - \phi_p)(dT/dx)_f + k_p\phi_p(dT/dx)_p}{\phi_p(dT/dx)_p(1 - \phi_p)(dT/dx)_f},
\]

where \((dT/dx)_f\) is the temperature gradient within the fluid and \((dT/dx)_p\) is the temperature gradient through the particle. The Maxwell model [74] was one of the first models proposed for solid–liquid mixtures with relatively large particles. It was based on the solution of the heat conduction equation through a stationary random suspension of spheres. The effective thermal conductivity is given by:

\[
k_{eff} = \frac{k_p + 2k_f(1 - \phi_p - k_f)}{k_p + 2k_f - \phi_p(k_p - k_f)k_f}
\]

\[
k_{eff} = k_f + \frac{3\phi_p(k_p - k_f)}{k_p + 2k_f - \phi_p(k_p - k_f)k_f}.
\]
where $k_p$ is the thermal conductivity of the particles, $k_f$ is the fluid thermal conductivity, and $\phi_p$ is the volume fraction of the suspended particles. Maxwell model is accurate to order $1/\phi_p$ and applicable to $\phi_p \ll 1$ or $\phi_p - 1 \ll 1$. Bruggeman [75] proposed a model to study the interactions between randomly distributed spherical particles. The Bruggeman model can be expressed as:

$$k_{\text{eff}} = k_p + \frac{(3\phi_p - 1) k_p}{k_f} + \frac{(3(1 - \phi_p) - 1) + \sqrt{3}}{4} \cdot \frac{k_p}{k_f}.$$  

The Bruggeman model [75] is applicable for large volume fraction of spherical particles. For low volume fractions, the Bruggeman model [75] results in approximately the same results as the Maxwell model [74]. For non-spherical particles, Hamilton and Crosser [73] developed a model for the effective thermal conductivity of two-component mixtures. Their model was a function of the thermal conductivity of both the base fluid and the particle, volume fraction of the particles, and the shape of the particles. For the thermal conductivity ratio of two phases larger than 100 ($k_p/k_f > 100$), the thermal conductivity of two-component mixtures can be expressed as follows [73]:

$$k_{\text{eff}} = \frac{k_p + (n - 1)k_f + (n - 1)\phi_p(k_p - k_f)k_f}{k_p + (n - 1)k_f - \phi_p(k_p - k_f)}.$$  

where $n$ is the empirical shape factor given by $n = 3/\psi$, and $\psi$ is the particle sphericity, defined by the ratio of the surface area of a sphere with volume equal to that of the particle, to the surface area of the particle. Maxwell's model [74] is a special case of Hamilton and Crosser's model for sphericity equal to one (for spheres: $n = 3$). The shape factor for cylinders is $n = 6$. Apparently, the most notable drawback of the Hamilton Crosser model [73] is that important physical parameters such as temperature and particle size are not considered. Tables 5–7 summarize some pertinent models for the effective thermal conductivity of nanofluids including the effects of Brownian motion and nano-layer.
enhancement varying from anomalously large to small values as over, published results demonstrate thermal conductivity enhancement of nanofluids have not been well understood. More- cations, the pertinent mechanisms of the thermal conductivity by the significant benefits of utilizing nanofluids in various appli- and temperature oscillation. Despite numerous studies motivated techniques such as transient hot wire, steady-state parallel plates, to measure the thermal conductivity of nanofluids using different

Table 6
Summary of theoretical models for the effective thermal conductivity of nanofluids (nano-layer effect).

<table>
<thead>
<tr>
<th>Models</th>
<th>Expressions</th>
<th>Physical model</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maxwell [74]</td>
<td>( k_{\text{eff}} = k_{n} + k_{p} + 2k_{p}f_{n}k_{p})</td>
<td>- Based on the conduction solution through a stationary random suspension of spheres</td>
<td>- Spherical particles - Accurate to order ( \phi_{p}^{3} ) - Applicable to high volume fraction of spherical particles</td>
</tr>
<tr>
<td>Bruggeman [75]</td>
<td>( k_{\text{eff}} = \frac{\left( k_{n} + 2k_{p}\phi_{p}^{3} + k_{p}\phi_{p}^{3}\right)}{4\phi_{p}^{3}} )</td>
<td>- Based on the differential effective medium (DEM) theory to estimate the effective thermal conductivity of composites at high particle concentrations</td>
<td>- Suspension with spherical inclusions - No shape factor - Spherical and non-spherical particles</td>
</tr>
<tr>
<td>Hamilton– Wasp [76]</td>
<td>( k_{\text{eff}} = \frac{k_{n}\phi_{p}^{3} + k_{p}\phi_{p}^{3} - k_{n} + k_{p}}{\phi_{p}^{3}} )</td>
<td>- Based on the effective thermal conductivity of a two-component mixture</td>
<td>- ( n = 3 ) (spheres), ( n = 6 ) (cylinders) - Special case of Hamilton and Crosser's model with ( n = 3 )</td>
</tr>
<tr>
<td>Jaffary [77]</td>
<td>( k_{\text{eff}} = k_{n} + 3\phi_{p}^{3} + \phi_{p}^{3}\left( 3\phi_{p}^{3} + \frac{2}{3} + \frac{3}{2} + \ldots \right) )</td>
<td>- Based on the conduction solution through a stationary random suspension of spheres</td>
<td>- High order terms represent pair interactions of randomly dispersed spherical particles - Accurate to order ( \phi_{p}^{3} ) - Accurate to order ( \phi_{p}^{2} )</td>
</tr>
<tr>
<td>Davis [78]</td>
<td>( k_{\text{eff}} = 1 + \frac{3\phi_{p}^{3} + \phi_{p}^{3}\left( 3\phi_{p}^{3} + \frac{2}{3} + \frac{3}{2} + \ldots \right) \phi_{p}^{3}}{1 + \phi_{p}^{3}} )</td>
<td>- Green's theorem was applied to the space occupied by the matrix material (spherical inclusions) - Decaying temperature field was used</td>
<td>- Spherical and non-spherical particles - Spherical particles: ( a = 2.25, b = 2.27 ) for ( \kappa = 10 ); ( a = 3, b = 4.51 ) for ( \kappa = \infty )</td>
</tr>
<tr>
<td>Lu and Lin [79]</td>
<td>( k_{\text{eff}} = 1 + \phi_{p}^{3} + b\phi_{p}^{2} + c\phi_{p} )</td>
<td>- The effective conductivity of composites containing aligned spheroids of finite conductivity was modeled with the pair interaction - The pair interaction was evaluated by solving a boundary value problem involving two aligned spheroids</td>
<td>- Spherical and non-spherical particles - Spherical particles: ( a = 2.25, b = 2.27 ) for ( \kappa = 10 ); ( a = 3, b = 4.51 ) for ( \kappa = \infty )</td>
</tr>
</tbody>
</table>

5.1. Experimental investigations

Several experimental studies were conducted in the literature to measure the thermal conductivity of nanofluids using different techniques such as transient heat wire, steady-state parallel plates, and temperature oscillation. Despite numerous studies motivated by the significant benefits of utilizing nanofluids in various applications, the pertinent mechanisms of the thermal conductivity enhancement of nanofluids have not been well understood. Moreover, published results demonstrate thermal conductivity enhancement varying from anomalously large to small values as seen in Table 8. Table 8 shows a comparison of the experimental thermal conductivity enhancements of Al2O3 nanofluids cited in the literature. Al2O3 and CuO are the most common nanoparticles used in the literature. Several studies have reported enhancement in the thermal conductivity of nanofluids at room temperature [84]. Fig. 6a and b show the effective thermal conductivity measurements at ambient temperature for Al2O3–water and CuO–water nanofluids at various volume concentrations and nanoparticle diameters. Fig. 9 shows that the effective thermal conductivity increases with an increase in the volume fraction. In addition, the size of the particles is found to have a significant effect on the thermal conductivity enhancement. It should be noted that smaller particles exhibit larger surface area – to – volume ratio than the larger particles. As such, smaller particle diameters can possibly result in a larger augmentation in the effective thermal conductivity. It is interesting to note from Fig. 6a and b that the Hamilton–Crosser model [73] may represent a good approximation for the effective thermal conductivity value for smaller volume fractions (\( \phi_{p} \leq 4\% \)).
### Table 7
Summary of theoretical models for the effective thermal conductivity of nanofluids (Brownian effect).

<table>
<thead>
<tr>
<th>Models</th>
<th>Expressions</th>
<th>Physical model</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wang et al. [19]</td>
<td>$k_{eff} = \left(1 - \phi_p\right)k_f + \phi_p k_p + 3C_{pr}^f k_p Re_C^f Pr_C^f \phi_p$</td>
<td>- Based on the effective medium approximation and the fractal theory for predicting the thermal conductivity of nanofluids</td>
<td>- Accounts for the size effect and the surface adsorption of nanoparticles</td>
</tr>
<tr>
<td>Xuan et al. [80]</td>
<td>$k_{eff} = \frac{1 - \phi_p + \phi_p \sqrt{2 - 3 \phi_p} k_f}{1 - \phi_p + \sqrt{2 - 3 \phi_p} \phi_p}$</td>
<td>- Based on Maxwell model and the diffusion-limited aggregation model are applied to simulate random motion and the aggregation process of the nanoparticles</td>
<td>- Includes the effect of random motion, particle size, concentration, and temperature</td>
</tr>
<tr>
<td>Jang and Choi [10]</td>
<td>$k_{eff} = k_f(1 - \phi_p) + \phi_p k_p + 3C_{pr}^f k_p Re_C^f Pr_C^f \phi_p$</td>
<td>- A theoretical model was developed based on kinetics, Kapitza resistance, and convection</td>
<td>- Considered four modes of energy transport: collision between fluid molecules, thermal diffusion of nanoparticles, collision between nanoparticles due to Brownian motion, and thermal interactions of dynamic nanoparticles with fluid molecules</td>
</tr>
<tr>
<td>Prasher et al. [81]</td>
<td>$k_{eff} = (1 + 50Re^{0.303} \phi_p) \sqrt{\frac{k_f + 2k_p + 3k_p}{k_f + k_p}} \frac{k_f}{k_p}$</td>
<td>- Based on Maxwell model and heat transfer in fluidized beds</td>
<td>- Accounts for convection caused by the Brownian motion from multiple nanoparticles</td>
</tr>
<tr>
<td>Koo and Kleinstreuer [72,82]</td>
<td>$k_{eff} = k_{static} + k_{Brownian} = \frac{k_f + 2k_p + 3k_p}{k_f + k_p} k_f + 5 \times 10^4 \phi_p k_p \phi_p \sqrt{\frac{k_f}{k_p}} f(T, \phi_p)$</td>
<td>- Based on Maxwell model and heat transfer in fluidized beds</td>
<td>- Considered surrounding liquid traveling with randomly moving nanoparticles</td>
</tr>
<tr>
<td>Chon et al. [83]</td>
<td>$k_{eff} = 1 + 6.47 \phi_p^{0.74} \frac{k_f}{k_p} \phi_p^{0.747} \sqrt{\frac{k_f}{k_p}} \frac{k_f}{k_p} \frac{Pr}{Re}^{0.9955} Re^{1.2321}$</td>
<td>- Curve fitting of the available experimental data to determine the effective conductivity due to Brownian motion</td>
<td>- Reynolds number is based on the Brownian motion velocity</td>
</tr>
</tbody>
</table>

### Table 8
Comparison of the experimental thermal conductivity enhancements of Al₂O₃ nanofluids cited in the literature.

<table>
<thead>
<tr>
<th>Base fluid</th>
<th>$D_p$ (nm)</th>
<th>$\phi_p$ (%)</th>
<th>%Thermal conductivity enhancement</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>38.4</td>
<td>4</td>
<td>9.4% (21 °C), 24.3% (51 °C)</td>
<td>Temperature Oscillation</td>
</tr>
<tr>
<td>Water</td>
<td>131 nm</td>
<td>4</td>
<td>24% (51 °C)</td>
<td>Steady-state parallel plates</td>
</tr>
<tr>
<td>Water</td>
<td>13 nm</td>
<td>4.4</td>
<td>33%</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>38.4 nm</td>
<td>4.3</td>
<td>11%</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>28 nm</td>
<td>5.5</td>
<td>16%</td>
<td>Steady-state parallel plates</td>
</tr>
<tr>
<td>Water</td>
<td>28 nm</td>
<td>5</td>
<td>24.5%</td>
<td>Steady-state parallel plates</td>
</tr>
<tr>
<td>Water</td>
<td>11 nm</td>
<td>1</td>
<td>14.8% (70 °C)</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>47 nm</td>
<td>1</td>
<td>10.2% (70 °C)</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>150 nm</td>
<td>1</td>
<td>4.8% (60 °C)</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>47 nm</td>
<td>4</td>
<td>28.8% (70 °C)</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>36 nm</td>
<td>6</td>
<td>28.2%</td>
<td>Steady-state parallel plates</td>
</tr>
<tr>
<td>Water</td>
<td>47 nm</td>
<td>6</td>
<td>26.1%</td>
<td>Steady-state parallel plates</td>
</tr>
<tr>
<td>Water</td>
<td>20 nm</td>
<td>5</td>
<td>15%</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>11 nm</td>
<td>5</td>
<td>8%</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>20 nm</td>
<td>5</td>
<td>7%</td>
<td>Transient hot wire</td>
</tr>
<tr>
<td>Water</td>
<td>40 nm</td>
<td>5</td>
<td>10%</td>
<td>Transient hot wire</td>
</tr>
</tbody>
</table>

*EG: ethylene glycol.*

A general correlation for the effective thermal conductivity of Al₂O₃–water and CuO–water nanofluids at ambient temperature accounting for various volume fractions and nanoparticles diameters is obtained by the present authors using the available experimental data in the literature. This model can be expressed as:

$$k_{eff} = k_f \left(1 + 1.0112\phi_p + 2.4375\phi_p^2 \left(\frac{47}{D_p[nm]}\right) \right) - 0.0248\phi_p \left(\frac{k_f}{0.613}\right);$$

where $k_f$ is the thermal conductivity of water. Fig. 7 demonstrates that the general correlation, represented by Eq. (20), is in good agreement with the experimental measurements of Al₂O₃–water and CuO–water nanofluids.

Thermal conductivity measurements at different temperatures are essential because the measurements at ambient temperature are not adequate for estimating the heat transfer characteristics. Fig. 8 shows a comparison of the relative effective thermal conductivity (ratio of the effective thermal conductivity of the nanofluid to the thermal conductivity of the base fluid at the same temperature) results of Al₂O₃–water nanofluid obtained from various experimental results as a function of volume fraction and nanoparticle’s diameter. Fig. 8 shows that temperature has a significant
A general correlation is developed for Al₂O₃–water nanofluid by the present authors using the available experimental data at various temperatures, nanoparticle’s diameter, and volume fraction. The developed correlation is expressed in terms of nanoparticle’s diameter, volume fraction, dynamic viscosity of water, effective dynamic viscosity of the nanofluid, and temperature as follows:

\[
\frac{k_{\text{eff}}}{k_f} = 0.9843 + 0.398\phi_p^{0.7383}\left(\frac{1}{d_p(\text{nm})}\right)^{0.2246}\left(\frac{\mu_{\text{eff}}(T)}{\mu_f(T)}\right)^{0.0235}
- 3.9517\phi_p\frac{T}{2} + 34.034\phi_p^2\frac{T}{2} + 32.509\phi_p^{1.5} - 10\%
\]

\[\text{if } 11 \text{ nm} \leq d \leq 150 \text{ nm}, \text{ and } 20 \text{ °C} \leq T \leq 70 \text{ °C}.
\]

where the dynamic viscosity (Pa s) of water at different temperatures can be expressed as:

\[
\mu_f(T) = 2.414 \times 10^{-5} \times T^{24.8/(T-140)}.
\]

The validity of the above correlation (Eq. (21)) is depicted in Fig. 9. Fig. 9 shows a very good agreement between the predicted relative effective thermal conductivity by our model and the experimental data.

Different models were developed in the past for the effective thermal conductivity of a two-component mixture such as Hamilton–CROSSER model [73]. Although this model gave a good approximation for the effective thermal conductivity of the Al₂O₃–water and CuO–water nanofluids for small volume fractions at room temperature, it does not provide a good approximation of the effective thermal conductivity at various temperatures shown in Fig. 9 because this model as well as a number of other models in this area do not properly account for the variations of the effective thermal conductivity with temperature. Therefore, these analytical models cannot be used to determine the effective thermal conductivity of nanofluids at various temperatures. Instead, Eq.
developed in this work should be used. Table 9 summarizes all the correlations that were developed in the present work for nanofluids.

5.2. Natural convection heat transfer utilizing nanofluids

Conflicting results were reported in the literature regarding natural convection heat transfer enhancement using nanofluids. The findings of both experimental and analytical investigations are still in disagreement. Analytical studies show an increase in heat transfer with an increase in the volume fraction of nanoparticles which is not in agreement with the experimental results. Since Rayleigh number, which is the ratio of the buoyancy to the viscous forces, represents a significant parameter in natural convection processes, comparison of nanofluid Rayleigh number to the base fluid Rayleigh number at various volume fractions and temperature is highlighted in this section. Using a scale analysis approach, the viscous and buoyancy forces can be expressed as:

\[
\text{Viscous force} : \frac{\nu}{H^2}(\text{N/kg}),
\]

\[
\text{Buoyancy force} : \frac{g}{\Delta T}(\text{N/kg}).
\]

The ratio of nanofluid Rayleigh number to that of the base fluid can then be expressed as:

\[
\frac{R_{\text{nf}}}{R_{\text{f}}} = \frac{l_{\text{nf}}}{l_{\text{f}}} \frac{\nu_{\text{f}}}{\nu_{\text{nf}}} \frac{\alpha_{\text{f}}}{\alpha_{\text{nf}}}. \tag{24}
\]

Fig. 10a shows that the ratio of the Rayleigh number of nanofluid to that of the base fluid decreases with an increase in the Al₂O₃ volume fraction. Higher volume fractions of the solid nanoparticles causes an increase in the viscous force of nanofluids and consequently suppresses heat transfer. Moreover, Fig. 10a shows the effect of varying particle diameter on the Rayleigh number ratio. As the particle diameter increases, the ratio of the Rayleigh numbers decreases because the effective thermal conductivity of nanofluids decreases and the kinematic viscosity increases with an increase in the size of nanoparticles. However, the rate of
increase of the kinematic viscosity of the nanofluid with the particle size is larger than the resulting decrease of the effective thermal conductivity. This may provide a physical reason for the reduction in natural convection heat transfer enhancement with an increase in the volume fraction of nanoparticles at room temperature.

The effect of varying the temperature of nanofluids and volume fraction on the ratio of Rayleigh numbers is illustrated in Fig. 10b for nanoparticles diameter of 36 nm. Fig. 10b shows that the ratio of nanofluid Rayleigh number to the base fluid Rayleigh number increases with an increase in the temperature. Moreover, this ratio is higher for volume fraction of 1% compared to 4% for various temperatures. This is because the kinematic viscosity and the effective thermal conductivity of nanofluids increase with an increase in the volume fraction of nanoparticles. For volume fraction of 1%, Fig. 10b shows an interesting result associated with the fact that nanofluid Rayleigh number is smaller than the Rayleigh number of base water below 31 °C. For temperatures greater than 31 °C, Fig. 10b shows that nanofluid Rayleigh number is higher than that of the base water. Hence, nanofluids may exhibit natural convection heat transfer enhancement at high temperatures. This is associated with the behavior of the kinematic viscosity and the thermal diffusivity for both nanofluid and the base water at various temperatures as shown in Fig. 11. Hence, Fig. 10a and b shows that natural convection heat transfer is not exclusively characterized by the effective thermal conductivity of nanofluids but also depends on the viscosity of nanofluids.

5.3. Surface tension

Studies on surface tension of nanofluids are limited in the literature [92–96]. Golubovic et al. [92] showed experimentally that the surface tension of Al₂O₃–water nanofluid does not change for concentrations of nanoparticles considered in their work (0–0.01 g/l) and the surface tension is approximately equal to surface tension of pure water at T = 24 °C. Xue et al. [93] showed insignificant effect of carbon nanotube nanofluid on surface tension
Table 9
Summary of the correlations developed in the present work.

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Room temperature</th>
<th>Temperature dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density Specific hat</td>
<td>( \rho_{eff} = (1 - \phi_p) \rho_{f} + \phi_p \rho_p )</td>
<td>( \rho_{eff} = 1001.064 + 2738.6191 \phi_p - 0.209570 \phi_p \leq 0.04.5 \leq T(\text{C}) \leq 40 ) N/A</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>( \beta_{eff} = \frac{(1 - \phi_p) \beta_{f} + \phi_p \beta_p}{\rho_f} )</td>
<td>( \rho_{eff} = (\rho_f - 0.479 \phi_p + 9.3158 \times 10^{-3} T - 2.2211) \times 10^{-3} )</td>
</tr>
<tr>
<td>Viscosity</td>
<td>( \mu_{eff} = (1 - \phi_p) \mu_{f} + \phi_p \mu_p ) N/A</td>
<td>( \mu_{eff} = (\mu_f - 0.0491 + 2.2217 \times 0.574 \phi_p - 0.1634 \phi_p^2 + 23.053 \phi_p^3 + 0.0132 \phi_p^4) )</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>( \frac{k_{tot}}{k_f} = 1.0 + 1.0112 \phi_p + 2.4375 \phi_p \left( \frac{\delta}{\sigma \pi / d} \right) - 0.0248 \phi_p \left( \frac{\delta}{\sigma \pi / d} \right)^2 )</td>
<td>( \frac{k_{tot}}{k_f} = 0.9843 + 0.398 \phi_p^{0.2318} \left( \frac{\delta}{\sigma \pi / d} \right)^{0.0235} \left( \frac{\delta}{\sigma \pi / d} \right)^{0.0215} )</td>
</tr>
</tbody>
</table>

\( \frac{d_p}{d} = 36 \text{ nm} \)

\( \frac{d_p}{d} = 47 \text{ nm} \)

Fig. 10. Effect of volume fraction and temperature on the ratio of the Rayleigh numbers for different particle diameters (Al₂O₃–water nanofluid) (a) room temperature; (b) various temperatures.
compared with pure water. Similarly, Kim et al. [94] found that the surface tension of Al₂O₃–water nanofluids (0.1% volume fraction) differed negligibly from those of pure water. The effect of temperature on the surface tension of nanofluids was studied by Murshed et al. [95]. Their experimental results showed that nanofluids having TiO₂ nanoparticles of 15 nm diameter in deionized water exhibit substantially lower surface tension than those of the base fluid (i.e. deionized water). Similarly, Zhu et al. [96] showed that the surface tension of nanofluids was highly dependent on the temperature. One can note from the above that as expected temperature plays a significant role on the surface tension of nanofluids.

5.4. Nucleate pool boiling and critical heat flux (CHF) of Nanofluids

Several experimental studies on the nucleate pool boiling and CHF characteristics of nanofluids have been investigated in the literature. Many researchers expected that the addition of nanoparticles would have a potential to enhance the boiling heat transfer characteristics. Conflicting results were reported on the effect of the nanoparticles concentration on the pool boiling characteristics of nanofluids. Some studies have shown that the addition of nanoparticles exhibited a decrease or no change in the nucleate boiling heat transfer rate [20,21,97–99] while other studies have illustrated an increase [100–103]. However, most CHF experiments using nanofluids were in agreement in reporting an enhancement in CHF for pool boiling conditions [98,99,104,105]. The mechanism for the CHF enhancement may be attributed to the deposition and sintering of nanoparticles on the boiling surfaces resulting in an increase in the nucleation sites [98]. For example, Kim et al. [94] conducted an experimental study on the CHF characteristics of nanofluids in pool boiling. Their results show that the CHF of nanofluids containing TiO₂ or Al₂O₃ were enhanced up to 100% over that of pure water.

You et al. [98] conducted an experimental study to determine the boiling curve and the CHF at pool boiling conditions from a flat square polished copper heater immersed in Al₂O₃–water nanofluid at various volume fractions ranging between 0.0 g/l and 0.05 g/l. Their results illustrate a drastic enhancement in the CHF for Al₂O₃–water nanofluids (~200% higher than pure water; measured in nanofluids containing 0.005 g/l of Al₂O₃ nanoparticles). However, heat transfer enhancement or degradation was not observed in the nucleate boiling regime for nanofluids. For example, Vassallo et al. [106] experimentally demonstrated a marked increase in the CHF for both nano- and micro-solutions (silica–water) at the same concentration (0.5% volume fraction) compared to the base water. However, from boiling curves data, no heat transfer enhancement of nanofluids was observed in the nucleate boiling regime.
5.5. Nucleate pool boiling heat transfer and CHF mechanisms of nanofluids

Several studies have been conducted in the literature to explore the enhancement mechanisms or deterioration of nucleate pool boiling heat transfer using nanofluids. These mechanisms include formation of nanoparticles coatings on the surface during pool boiling of nanofluids [99], decrease in active nucleation sites due to nanoparticle sedimentation on the boiling surface [107], and the wettability change of the surface [20,21]. The available experimental results on nucleate pool boiling heat transfer coefficient of nanofluids are in disagreement. While the CHF enhancement results by nanofluids are consistent in the literature, the responsible mechanisms are not verified. Golubovic et al. [92] concluded that the major reason behind the increase of CHF in pool boiling of nanofluids is a decrease in the static surface contact angle.

Many other studies consider the primary reason for CHF enhancement is due to the surface coating effect [99,100,106,108–111]. For example, Bang and Chang [99] conducted an experimental study on boiling heat transfer characteristics of nanofluids with nanoparticles suspended in water using different concentrations of alumina nanoparticles. The CHF performance was enhanced for both horizontal (32%) and vertical (13%) flat surfaces and the authors related this enhancement to a change of surface characteristics by the deposition of nanoparticles. If this reasoning is accepted, it might be easier to modify the boiling surface in pursuit of a greater number of nucleation sites per area rather than using nanofluids. The CHF enhancement is normally achieved by increasing boiling surface area using a variety of fin shapes and sizes [112,113]. Anderson and Mudawar [112] illustrated that the surfaces with microgrooves and square microstuds are highly effective in enhancing the nucleate boiling heat transfer coefficient in Fluorinert electronic liquid (FC-72) and increasing CHF values by up to 2.5 times compared to a smooth surface. Honda et al. [114] and Wei et al. [115] showed that CHF values for the nano-roughened surface and micro-pin-finned surfaces were respectively 1.8–2.2 and 2.3 times those for a smooth silicon surface. Ujereh et al. [116] performed experiments to assess the impact of coating silicon and copper substrates with nanotubes on pool boiling performance. Fully coating the substrate surface with carbon nanotubes was found to be highly effective at reducing the incipience superheat and significantly enhancing both the nucleate boiling heat transfer coefficient and CHF.

More robust physical models are essential to explain the effect of nanofluids on nucleate pool boiling and CHF. The thermophysical properties of nanofluids such as surface tension, density, viscosity, specific heat, etc.; may also have an effect on nucleate pool boiling heat transfer coefficient and CHF. Detailed knowledge of the thermophysical properties of nanofluids, structure of the boiling surface, and coating of nanoparticles can be helpful in resolving the controversies in the pool boiling heat transfer coefficient of nanofluids as well as in illustrating the mechanisms that cause the significant increase in CHF.

6. Conclusions

Thermophysical characteristics of nanofluids and their role in heat transfer enhancement are analyzed in this work. General correlations for the effective thermal conductivity and viscosity of nanofluids are developed based on the pertinent experimental data in terms of the volume fraction, particle diameter, temperature, and the base fluid physical properties. The effective viscosity of nanofluids is found to increase with an increase in the volume fraction and decrease with an increase in the temperature. The effective thermal conductivity of nanofluids increases with an increase in temperature and volume fraction and decreases with an increase in the particle diameter. At room temperature, classical models can be used to estimate the thermal conductivity and viscosity of nanofluids for low volume fractions. However, these models cannot predict the thermal conductivity at other temperatures. Correlations based on available experimental data accounting for the temperature effect are developed in this work. The findings from experimental and analytical investigations are still in disagreement regarding natural convection enhancement utilizing nanofluids. The current work illustrates that the viscosity of nanofluids plays a key role in predicting the heat transfer characteristics. Differences in thermal conductivity and viscosity measurements in the literature using different measurement techniques are highlighted. Moreover, for high heat flux applications, the experimental results illustrate conflicting results in pool boiling heat transfer characteristics while the critical heat flux of nanofluids shows a significant increase with the addition of nanoparticles. While the addition of nanoparticles appear not to have a significant effect on the surface tension of the nanofluid, experiments show that the surface tension of the nanofluids is significantly dependent on the temperature. Several pertinent correlations for the thermophysical properties of nanofluids were developed in this work based on the available experimental data. This work clearly illustrates the need for additional investigations in measuring the nanofluid properties.

References


