Heat transfer characteristics and CHF prediction in nanofluid boiling

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ABSTRACT

A composite model for the average heat flux using both pure fluids and nanofluids is established incorporating microlayer evaporation, transient conduction due to bubble departure, and microconvection due to bubble growth and movement. Our model also takes into account bubble influence area interference. The average heat flux and the critical heat flux (CHF) when the surface dryout fraction exceeds a certain value are predicted. Nanofluid properties were calculated and the contact angles obtained from experiments were utilized in the present model. The analytical heat flux and CHF values using both regular fluids and nanofluids were compared with the experimental results and existing theories and were found to be in close agreement. It was found that CHF enhancement value increases to 3.54 as the contact angle decreases from 80° to 20°, which is in agreement with recent experimental results.

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1. Introduction

The heat transfer characteristics and thermal properties of nanofluids have been investigated more intensely in the past couple of decades for different heat transfer applications by various researchers [1–3]. Water is the most popular base fluid, while glycol–ethylene, oil, bio-fluids, and polymer solutions have also been used. A variety of materials are commonly used as nanoparticles, such as alumina, titania, silica, zirconia, graphite, carbon nanotubes, diamond, copper and gold. [4]. In addition to single-phase heat transfer applications, nanofluid is also used in two-phase heat transfer applications.

CHF is found to be substantially enhanced with nanofluid boiling, while contradictory results with respect to the average wall heat transfer rates have been reported [5,8–15]. Das et al. [5] experimentally studied pool boiling of alumina-water nanofluid and observed deterioration of the average heat flux. The nanoparticle size range varied from 50 to 150 nm, and the mean diameter was 38 nm. The concentration of the liquid varied from 1% to 4%. They argued that the nanoparticles, which were much smaller than the surface roughness (0.2–1.2 μm), deposit on the relatively uneven surface during boiling and hence created a rougher surface. This causes degradation of the boiling characteristics and the surface smoothing overshadows the thermal conductivity enhancement of the nanofluids. Coursey and Kim [6] employed water and ethanol based nanofluids with aluminum nanoparticle concentrations from 0.001 to 10 g/L. Nanofluids were found to degrade or have little influence on the boiling performance. Larger concentrations (≥0.5 g/L) resulted in a moderate (up to 37%) increase in the CHF.

The main reason for the deterioration of heat flux is assumed to be the deposition of nanoparticles on the surface which changes the surface wettability. Kim et al. [7] experimentally studied alumina, silica, and zirconia based nanofluids and found that when the concentration was smaller than 0.1% by volume, CHF could be greatly enhanced due to the buildup of a porous layer of nanoparticles on the surface.

The nanofluid boiling experimental results [5,8–20] are summarized in Table 1. In this Table, nanoparticle size and concentration is specified under the column entitled “nanofluid properties”. The heater properties, the contact angle and roughness characteristics are listed under the column entitled “surface property”. The average boiling heat flux (BHF), CHF, the boiling heat transfer coefficient (BHTC) as well as the maximum deterioration and enhancement ratios values at specified nanofluid concentrations are also illustrated in Table 1.

Although pool boiling with nanofluids has been investigated extensively experimentally, there are very few reports on an analytical study of nanofluid boiling. Models on nanofluid boiling are needed. Compared to boiling with pure liquid, nanofluid boiling involves the change of liquid thermal properties and the movement of particles. Experiments show that at low concentrations, changes in liquid thermal properties is not significant [12,14,21,22]. Nanoparticles deposited on the surface alter the surface conditions [23] and the surface contact angle [8,11–14]. Surface contact angle is...
an important characteristic and it can substantially influence the active nucleation sites [25]. In our analysis, we will use the contact angle as a parameter to define surface characteristics.

A number of researchers have proposed several theories on the boiling process and the associated heat transfer [26–31]. However, the literature lacks a dominant model representation of boiling heat flux. The earliest researchers simplified the boiling process to a single-phase convection process, including Rohsenow [26], Tien [27] and Zuber [28]. Forster and Greif [29] proposed that departing bubble worked as a micropump to push hot liquid away and cold liquid in. Mikic and Rohsenow [31] postulated that when bubbles depart, they take away the superheated liquid layer adjacent to the heating surface and cold liquid rushes in and transient conduction occurs.

Different mechanisms are active during different bubble periods. Judd and Huang [32] first combined microlayer evaporation, transient conduction and natural convection to set up a comprehensive model for boiling heat transfer. Benjamin and Balakrishnan [33] proposed a similar model and accounted for the activation period for transient conduction and microlayer evaporation. These two models considered the situations at low to moderate heat flux regions where bubbles did not interfere with each other. Since the boiling heat transfer attributes have not been fully understood, more work needs to be done to better reveal the heat transfer mechanism in nucleate boiling.

Critical heat flux (CHF) is also an important characteristic in boiling. CHF is the end of nucleate boiling and refers to the limit of safe operation of a heat transfer system. In nuclear reactors or electrical components, loss of liquid contact with surface will dramatically decrease the heat transfer coefficient and causes damage to the surfaces. To investigate CHF and its mechanism, a number of theories have been proposed to predict the critical heat flux.

Zuber [34] proposed that Taylor and Helmholtz instabilities were responsible for the occurrence of CHF. As the heat flux increases, the velocity of vapor jets above the nucleating bubbles increase dramatically, occupying the space of liquid between the jets toward the surface thus initiating an instability process. Zuber’s correlation is the most widely used to predict pool boiling CHF as:

\[ q_{\text{CHF}} = 0.131 \rho^0.8 \frac{h_g}{C_1} \left[ \sigma g \left( \rho_f - \rho_v \right) \right]^{0.25} \]  

Haramura and Katto [35] proposed a hydrodynamic model, that is, as the heat flux exceeds a certain value, a vapor blanket forms on the liquid film preventing bulk liquid from feeding the film resulting in the dryout of the liquid film thus substantially enhancing occurrence of CHF.

Kandlikar [36] took the effect of dynamic receding contact angle into consideration and developed a theoretical model to predict the CHF in saturated pool boiling. As the surface heat flux
increases, the force resulting from the evaporation at the bubble interface exceeds the gravitational and surface tension forces that are holding the bubble, resulting in the spread of vapor bubble over the surface causing CHF to occur. The following correlation by Kandlikar [36] predicts the effect of surface orientation angle and system pressure on CHF:

$$ q_{CHF}^2 = \rho_f h_{fg} \left( \frac{1 + \cos \theta}{16} \right) $$

$$ \times \left[ \frac{2}{\pi} \log \left( \frac{1}{1 + \cos \theta} \right) \frac{\cos \phi}{\cos \phi} \right] \left[ g \sigma (\rho_f - \rho_l) \right]^{1/4} \quad (2) $$

where \( \theta \) is the receding contact angle and \( \phi \) is the heater surface angle. The above equation predicts a continuous increase of CHF with decreasing surface contact angle.

The present work incorporates the main mechanisms in the boiling process and extends the analysis to high heat flux region where bubble influence areas interfere with each other. The CHF value can also be predicted by the present model. The obtained analytical results were compared with recent experimental results and classical equations and were found to be in good agreement.

2. Nanofluid properties

The density of a nanofluid can be expressed based on the mixture rule as:

$$ \rho_f = (1 - \phi) \rho_1 + \phi \rho_p $$

Similarly, specific heat and thermal expansion coefficient of nanofluid can be expressed [37] as:

$$ c_f = \left( 1 - \frac{\phi}{\rho_f} \right) c_1 + \frac{\phi}{\rho_f} c_p $$

$$ \beta_f = \left( 1 - \frac{\phi}{\rho_f} \right) \beta_1 + \frac{\phi}{\rho_f} \beta_p $$

The dynamic viscosity can be expressed as [38]:

$$ \mu_f = \left( 1 + 2.5 \phi + 6.2 \phi^2 \right) \mu_l $$

and the thermal conductivity can be expressed based on the correlation proposed by Khanafer and Vafai [37]:

$$ k_f = k_l \left[ 1.0 + 1.0112 \phi + 2.4375 \phi \left( \frac{47}{d_l (\text{nm})} \right) - 0.0248 \phi \left( \frac{k_l}{0.613} \right) \right] $$

3. Model development

Boiling is a complex phenomenon as it involves both phase change and forced convection due to bubble movement. A number of researchers have divided the analysis of pool boiling heat transfer into different parts [32,33]. In prior works, microlayer evaporation and transient conduction during bubble waiting time were analyzed in detail. However microconvection has not received much attention. In the present work, the total heat flux is divided into 4 parts: Microlayer evaporation, transient conduction under bubble, transient conduction after bubble departs, and enhanced natural convection. The interference of bubble influence areas was also analyzed in the present work.

### 3.1. Total bubble heat transfer in one bubble ebullition cycle

#### 3.1.1. Microlayer evaporation

Microlayer evaporation is considered to be an important mechanism during the bubble growth phase. The evaporation of this thin liquid layer under the bubble generates the vapor needed for the bubble growth as shown in Fig. 1a. However, due to the very small size of the microlayer, it is difficult to directly observe and measure it. An analytical model is presented here to predict the microlayer shape and the heat transfer rate. The microlayer is considered within a cylindrical coordinate system. The continuity equation for the microlayer can be expressed as:

$$ \frac{1}{r} \frac{\partial}{\partial r} \left( ru_r \right) + \frac{\partial u_y}{\partial y} = 0 $$
The radial momentum equation in the microlayer assumes the advection term is negligible and that the velocity gradient in the radial direction is much smaller than the velocity gradient in the vertical direction, so the lubrication approximation can be invoked to arrive at:

$$\frac{\partial P_f}{\partial r} = \frac{\mu}{r} \frac{\partial^2 u_f}{\partial y^2}$$  \hspace{1cm} (9)

The velocity at $y = 0$ and the shear stress at the interface is zero, so the boundary conditions can be expressed as:

$$u_f(y = 0) = 0, \quad \frac{\partial u_f}{\partial y} (\delta) = 0$$  \hspace{1cm} (10)

Integrating Eq. (9) subject to the boundary conditions in Eq. (10), results in the following expression for the velocity along the microlayer:

$$u_f = -\frac{1}{\mu} \frac{\partial P_f}{\partial r} \left( \frac{\delta y - \delta^3}{Z} \right)$$  \hspace{1cm} (11)

Integrating Eq. (8) and solving for $v_f$ gives:

$$v_f(\delta) - 0 = -\frac{1}{r} \frac{\partial}{\partial r} \int_0^\delta u_f dy$$  \hspace{1cm} (12)

Substituting Eq. (11) in Eq. (12), the vertical velocity can be expressed as:

$$v_f(\delta) = \frac{3}{2\mu} \frac{\partial}{\partial r} \left( \frac{\partial P_f}{\partial r} \right)$$  \hspace{1cm} (13)

Since the microlayer is quite thin, the velocity due to bubble evaporation perpendicular to the bubble interface is essentially vertical:

$$v_f(\delta) = \frac{q_f}{\rho_v h_{fg}}$$  \hspace{1cm} (14)

Eqs. (13) and (14) can be combined and rearranged as:

$$\frac{3\mu q_f}{\rho_v h_{fg} \delta^3} \frac{d}{dr} \left( \frac{dP_f}{dr} \right) = \frac{dP_f}{dr} + \frac{r}{\delta} \frac{d^2 P_f}{dr^2}$$  \hspace{1cm} (15)

The heat flux through the microlayer is based on Fourier's law,

$$q'' = k_f \left( \frac{T_w - T_{int}}{\delta} \right)$$  \hspace{1cm} (16)

While the modified Clausius–Clapeyron equation [39] for the heat transfer across the interface can be written as:

$$q'' = h_{ev} \left[ T_{int} - T_v + \frac{(P_f - P_v)T_v}{\rho_v h_{fg}} \right]$$  \hspace{1cm} (17)

As such, the interface temperature $T_{int}$ can be obtained by combining Eqs. (16) and (17) as:

$$\left( k_f + h_{ev} \right) T_{int} = \left[ k_f T_w + h_{ev} T_v - h_{ev} \frac{(P_f - P_v)T_v}{\rho_v h_{fg}} \right]$$  \hspace{1cm} (18)

Pressure drop across the interface is given by the modified Young–Laplace’s equation, obtained by a force balance which includes the surface tension, disjoining pressure, and the vapor recoil force:

$$P_f = P_v - \sigma \kappa - \frac{A}{\delta^3} + \frac{q''^2}{\rho_v h_{fg}}$$  \hspace{1cm} (19)

where $A$ is the Hamaker constant and is chosen as $10^{-21}$ based on neglecting the recoil pressure term in Eq. (19). This results

$$P_f = P_v - \sigma \kappa - \frac{A}{\delta^3}$$  \hspace{1cm} (20)

In the dryout region, $q'' = 0$ and $T_{int} = T_w$. Substituting for a zero curvature and heat flux in the above equation, the thickness of the dryout area is found to be:

$$\delta_0 = \frac{A}{(\frac{\mu}{\nu} - 1) h_{fg} \rho_v}$$  \hspace{1cm} (21)

The interfacial curvature is calculated as:

$$\kappa = \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{r \frac{\partial}{\partial r} \delta}{\sqrt{1 + (\frac{\partial}{\partial r})^2}} \right]$$  \hspace{1cm} (22)

Calculating the derivative of $P_f$ in Eq. (19) results

$$\frac{dP_f}{dr} = -\sigma \kappa' + 3 \frac{A}{\delta^3}$$  \hspace{1cm} (23)
and the second derivative is also calculated to be:

$$\frac{d^2 P_j}{dr^2} = -\sigma \kappa'' + \frac{3A}{\delta^2} \kappa'' - \frac{12A}{\delta^2} \delta''$$

Combining Eqs. (15), (23) and (24) to replace the first and second derivatives of $P_j$, results in an expression for the second derivative of the curvature:

$$F_1(\delta) = \kappa'' = \left( -\frac{3\pi q''}{\rho_l h_{lg}} - \frac{\sigma \kappa'}{r} + \frac{3A}{\delta^2} \delta' - \frac{12A}{\delta^2} \delta'' \right) / \sigma$$

Calculating the second derivative of $\kappa$ from Eq. (22) results:

$$\frac{d^2 \kappa}{dr^2} = \frac{2\delta'}{r^2 (1 + \delta^2)^2} + \frac{r \delta'' - 2\delta' \delta''}{r^2 (1 + \delta^2)^2} + \frac{3 \delta' \delta'' - 3r \delta'' - 9r \delta' \delta''}{r^2 (1 + \delta^2)^2}$$

$$\frac{15 \delta' \delta''}{r^2 (1 + \delta^2)^2} = F_2(\delta) + \frac{w''}{(1 + \delta^2)^2}$$

Combining Eqs. (25) and (26), the fourth derivative of $\delta$ can be obtained as:

$$\delta'' = [F_1(\delta) - F_2(\delta)](1 + \delta^2)^2$$

The slope of the microlayer at $R_{\text{max}}$ is $\tan(\theta)$ and the radius of the outer edge of the microlayer is $R \sin(\theta)$ hence the boundary conditions at the microlayer can be presented as:

$$\delta(R_0) = \delta_0, \quad \delta'(R_0) = 0$$

$$\delta'(R_{\text{max}}) = \tan(\theta), \quad R_{\text{max}} = \frac{D}{2} \sin(\theta)$$

### 3.1.2. Transient conduction after bubble departs

Experiments have shown that the transient conduction is the dominant heat transfer mode in plate pool boiling [40, 41]. Mikic and Rohsengow [31] were the first to propose a model and provide an expression representing transient conduction during the bubble waiting time. Their model assumes that when a bubble departs from the surface, it carries away the liquid with a radius of a bubble departure diameter, while the cold liquid rushes towards the surface. This process is illustrated in Fig. 1b. The heat transfer from the heating surface to the bulk liquid is considered as transient conduction from a hot surface to a semi-infinite body, which is the bulk liquid in this case.

The instantaneous heat flux at any time is expressed as:

$$q''_n(t) = \frac{k_l \Delta T}{\sqrt{\pi \xi t}}$$

In this work, the transient conduction $q''_n(t)$ is taken to be three times that of the growth time $t_0$ [33]. Integrating Eq. (29) over the time period $t_0$ and an influence area of four times the bubble projecting area as suggested by various researchers [31–33], the total heat transfer during the waiting period at low heat flux region can be obtained as:

$$Q_{\text{tot}} = \int_{t_0}^{t} q''_n dt \cdot \pi D^2 = \int_{t_0}^{t} k_l \Delta T dt \cdot \pi D^2 = \frac{3\pi k_l \Delta T D^2}{\sqrt{\pi \xi t}}$$

Eq. (30) is valid only when the influence areas do not interfere with each other. To take into account the bubble influence area interference, a dryout fraction parameter $F$ is defined as the ratio of bubble projecting area to the entire heating surface as:

$$F = \frac{\pi D^2}{4} \frac{N_{sd}}{N_{sd}}$$

When $\pi D^2 \leq 1/N_{sd}$, the bubble influence area is smaller than the average area for each bubble on the heating surface and the bubble interfaces are considered not to overlap. For this condition, the dryout fraction $F$ is less than 25% according to Eq. (31). When the dryout fraction exceeds 25%, the influence areas will be overlapping. The overlapping area is a function of the nucleation site density and the bubble influence area. The total overlapping area over the heating surface within a unit area (1 m²) is:

$$A_{\text{total}} = \pi D^2 \cdot \frac{N_{sd}}{N_{sd}} - 1$$

For each bubble, the average overlapping area can be approximated as the total overlapping area divided by the number of active nucleation sites in one unit area. Since each overlapping area is shared by two interfering bubbles, the average overlapping area for each bubble can be represented as:

$$A_{\text{overlap}} = \frac{1}{2} \left( \pi D^2 - \frac{N_{sd}}{N_{sd}} \right)$$

Therefore, the non-overlapping area for each bubble is:

$$A_{\text{non-overlap}} = 1/N_{sd} - \frac{1}{2} \left( \pi D^2 - \frac{N_{sd}}{N_{sd}} \right) = 3/2N_{sd} - \frac{1}{2} \pi D^2$$

The effect of overlapping of more than three bubbles was considered to be balanced by a heat transfer augmentation of the overlapping area due to repeated quenching.

In the overlapping area, due to repeated quenching, the transient conduction heat flux is larger than the heat flux at non-overlapping areas. This enhancement ratio was determined by considering the two interfering bubble influence areas, with each bubble departing and entering during the waiting period. The enhancement ratio is defined as:

$$K_{\text{overlap}} = \frac{\int_0^t q''_n dt + \int_0^t q''_n dt}{\int_0^t q''_n dt} = \sqrt{\frac{3 + 1}{\sqrt{3}}} = 1.58$$

So in the high heat flux region, the total heat transfer after bubble departs from the surface is:

$$Q_{\text{lc}} = \sqrt{\frac{k_l \Delta T}{\pi \xi f}} \left( K_{\text{overlap}} \cdot A_{\text{overlap}} + A_{\text{non-overlap}} \right)$$

$$= \sqrt{\frac{k_l \Delta T}{\pi \xi f}} \left( \frac{A_{\text{overlap}}}{2} \left( \pi D^2 - \frac{1}{N_{sd}} \right) + \frac{3}{2N_{sd}} \pi D^2 \right)$$

### 3.1.3. Transient conduction before the bubble departs

This mode of heat transfer occurs when the bubble contact line starts to shrink and the surface under the bubble is rewetted. The enhanced heat flux has been observed by various researchers [42–45]. The bubble starts to recede from $R_c$, which is the radius of the contact line. The rewetting process ends when the radius diminishes to zero as the bubble departs. The rewetting process before bubble departure is illustrated in Fig. 1c.

Considering when the liquid front approaches $R$ from $R_c$ ($0 < R < R_c$), the heat transfer rate at time $t_0$ in this rewetting process can be obtained by integrating the heat flux from the contact line to where the liquid front is encountered, resulting in

$$q(t_k) = \int_{t_k}^{t} k_l \Delta T \cdot \frac{2\pi dr}{\sqrt{\pi \xi (t - t_0)}}$$

Assuming the bubble contact line receding velocity, $u_c$, is constant, the total rewetting time is $t_k = R_c/u_c$, and the time when the contact line approaches $R$ is $t_k = (R - R_c)/u_c$. Substituting $t_k$ and $t_0$ into Eq. (37), the heat transfer rate at the moment $t_k$ is obtained as:
Integrating the heat transfer rate over the time period \( t_{re} \), the total heat transfer during the bubble rewetting period can be represented as:

\[
Q_{re} = \int_0^{t_{re}} q(t) \, dt = \frac{8 \pi k_b \Delta T u_c}{3 \sqrt{\pi} \mu} \left( R_a t_{re}^{1/2} - \frac{2}{3} u_c t_{re}^{1/2} \right)
\]  
(39)

The radius of the contact can be written as \( R_a = D/2 \sin(\theta) \). The rewetting time \( t_{re} \) is about half of the growth time \( t_g \) [43], and \( t_{w} = 3t_g \) is often considered in the nucleate boiling case [33], so \( t_{re} = 1/8t_g \).

Substituting \( t_{re} \) into Eq. (39), the heat transfer during the rewetting period can be expressed as:

\[
Q_{re} = \frac{8 \pi \Delta T R_a^2}{5 \sqrt{\pi}} \frac{t_{re}}{\sqrt{\pi f}} = 0.05 \frac{k_b R_a T_{re}^2 \sin^2 \theta}{\sqrt{\pi f}} \frac{1}{\sqrt{\pi}}
\]  
(40)

3.1.4. Microconvection and natural convection

The natural convection heat transfer coefficient over a horizontal flat plate can be expressed as:

\[
Nu = 0.31(Gr \cdot Pr)^{1/3}
\]  
(41)

\[
q_{nc}^i = h \cdot \Delta T = \frac{Nu \cdot k_f}{d} \Delta T
\]  
(42)

\[
Q_{nc} = q_{nc}^i \left( \frac{1}{N_{ad}} - \pi D_d^2 \right) \frac{1}{\sqrt{\pi}} = \frac{0.31(Gr \cdot Pr)^{1/3} \cdot k_f \left( \frac{1}{N_{ad}} - \pi D_d^2 \right)}{d} \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{\pi}} \Delta T
\]  
(43)

In nucleate boiling, the rapid bubble growth perturbs the liquid and enhances the natural convection. In addition to the bubble growth perturbing the liquid slightly, bubbles can also oscillate slightly during the bubble growth period. The natural convection is enhanced due to bubble growth, oscillation and departure. This part of enhanced convection around the bubble is referred to as microconvection and to evaluate its effect we utilize the local Nusselt number expression for forced convection over a flat plate.

The Reynolds number was calculated using Forster and Zuber’s definition [46] as:

\[
Re_b = \frac{\rho_b U_b D_d}{\mu}
\]  
(44)

The bubble growth velocity can be written as:

\[
R_b = Ja \left( \frac{2T}{\pi} \right)^{1/2}, \quad U_b = dR_b/dt
\]  
(45)

where \( Ja = \rho_b \sigma(T_w - T_{sat})/(\rho L_h) \). Substituting Eq. (45) in Eq. (44), the Reynolds number can be represented as:

\[
Re_b = \frac{\rho_b}{\mu} \left( \frac{\Delta T c_p D_d}{\mu_0 \nu \sqrt{\pi}} \right) \frac{1}{\sqrt{\pi}}
\]  
(46)

The Nusselt number due to microconvection is expressed based on laminar flow over a flat plate by

\[
Nu_{nc} = C_1 Re_b^\alpha (Pr)^{1/3}
\]  
(47)

where \( C_1 = 0.332 \) as is the case for the forced convection over a flat plate, and \( m = 0.67 \) as in Rohsenow’s model [26].

For the low heat flux region, where bubble influence areas do not interfere with each other, the total area of the microconvection is considered to be within the influence area and outside the bubble contact area and the total time \( t_{re} \) is 1/4 of the total ebullition cycle. As such, the microconvection can be expressed as:

\[
Q_{nc} = 0.332 Re_b^\alpha (Pr)^{1/3} \cdot \frac{k_f}{D_d} \Delta T \left( \frac{\pi D_d^2 \sin^2 \theta}{4} \right)
\]  
(48)

If the nucleation site density is too large, the bubble influence areas will interfere with each other. Considering the overlapping area will be influenced by two independent bubbles, microconvection over this area will be repeated, hence the enhancement ratio can be considered to be doubled. As such the total microconvection can be expressed as:

\[
Q_{nc} = \frac{Nu_{nc} \cdot k_f}{D_d} \Delta T
\]  
(49)

\[
Q_{nc} = \frac{1}{4} \left( 2q_{nc}^i \tilde{A}_{overlap} + q_{nc}^\prime \tilde{A}_{non-overlap} \right) \left( \frac{\pi D_d^2 \sin^2 \theta}{4} \right)
\]  
(50)

3.2. Nucleation site density

The active nucleation sites \( N_{ad} \) can be predicted using Hibiki and Ishii’s equation [47].

\[
N_{nc} = \frac{1 - \exp \left( - \left( \frac{\theta^2}{8 \mu f} \right) \right) \left\{ \exp \left( f(\rho^*) \frac{\nu}{R_c} \right) - 1 \right\}}{1 - \exp \left( f(\rho^*) \frac{\nu}{R_c} \right) - 1},
\]  
(51)

where \( N_{nc} = 4.72 \times 10^5 \) sites/m\(^2\), \( \mu = 0.722 \) rad, and \( \nu = 2.50 \times 10^{-6} \) m, \( R_c = 2 \sigma \left( 1 + (\rho_f/\rho) \right) / \rho_f \exp \{ h_g (T_g - T_{sat}) / (R_c T_{sat}) \} - 1 \),

\[
f(\rho^*) = -0.01064 + 0.48246 \rho^* - 0.22712 \rho^*^2 + 0.05468 \rho^*^3
\]  
(52)

and \( \rho^* = \Delta \rho/\rho_f \), \( \rho^* = \log(\rho^*) \) and \( R_c \) is the gas constant based on the molecular weight. This correlation predicts a decrease in active nucleation sites with a decrease in the contact angle. In nanofluid boiling, the nanoparticles deposit on the surface and tend to decrease the surface contact angle by filling in the cavities and smoothing the surface. The nucleation site density variations with contact angle change agree with the nanofluid boiling experiments.

3.3. Bubble departure frequency

Bubble departure diameter is obtained using a relationship provided by Carey [48]:

\[
D_d = \frac{1.65d_t \sigma}{g (\rho_f - \rho_c) \left( \frac{15.6 \rho_f}{g (\rho_f - \rho_c)} \right)^{1/3} \left( \frac{\rho_l k_f (T_w - T_{sat})}{h_g \rho_c} \right)}
\]  
(54)

where \( d_t = 6.0 \times 10^{-3} \) mm and \( \beta_d = 6.0 \). The bubble departure frequency can be obtained using the bubble growth equation and can be expressed as:

\[
f = \frac{1}{\pi} \frac{Ja}{D_d} \frac{2}{D_d}
\]  
(55)

3.4. Average heat flux and CHF calculation

The total heat transfer in one bubble ebullition cycle can be obtained by adding the five heat transfer contributions. The average heat flux at the surface can be obtained as:

\[
q'' = \frac{Q_{nc} + Q_{re} + Q_{sc} + Q_{nc} + Q_{nc}}{f \cdot N_{ad}}
\]  
(56)
Over the low heat flux region, \( Q_{ce} \) and \( Q_{mc} \) are expressed using Eqs. (30) and (48) and for the high heat flux region, \( Q_{ce} \) and \( Q_{mc} \) are expressed using Eqs. (36) and (50). \( Q_{mc} \) is expressed in Eq. (43) for low to moderate heat flux while it becomes negligible over high heat flux region. \( Q_{mc} \) is expressed by the radial integration of \( q_{mc} \) given by Eq. (16) and \( Q_{ce} \) is expressed by Eq. (40).

Based on our model results, transient conduction is the dominant mode of heat transfer at higher superheats, accounting for nearly 70% of the total heat transfer at higher superheats while microconvective transfer is about 10% of the total heat transfer. Microlayer evaporation accounts for less than 10% of the total heat transfer while natural convection accounts for about 60% of the total heat transfer at very low superheats. However, this percentage decreases dramatically when the superheat increases.

The critical heat flux can also be obtained by our analytical model. When the dryout fraction \( F \) exceeds a certain value, most of the surface is occupied by vapor, thus retarding the liquid from the bulk pool region to flow down to wet the surface. Also, when the bubbles are too close together, they will begin to touch each other and coalesce resulting in a large vapor blanket that covers the surface. CHF is considered to occur when all the bubbles merge and the liquid cannot flow down to wet the liquid. According to Zuber [28], when

\[
N_u D_b^2 = 1 \quad (57)
\]

static bubble interactions will occur. Under this condition, the dryout fraction \( F \) is \( \pi/4 \) according to Eq. (31). As such, the heat flux at a dryout fraction of \( \pi/4 \) is considered to be CHF.

4. Results and discussion

4.1. Average heat transfer characteristics

Rohsenow’s correlation [26] is widely used to represent pool boiling as shown in Eq. (58).

\[
q^* = \frac{\mu_l h_{fg} \left( \theta^* \phi_l - \phi_s \right)}{\sigma} \left\{ \frac{C_f (T_w - T_{sat})}{Pr_f C_{fg} h_{fg}} \right\}^{1/2} \quad (58)
\]

For water, the exponent of \( Pr \), \( s \) is chosen as 1.0. Dimensionless factor \( C_f \) accounts for the particular combination of surface and liquid characteristics. Usually \( C_f \) can range from 0.0065 to 0.0215. The value 0.013 is often chosen for water–copper and water–platinum combination. Rohsenow’s correlation is widely used in engineering applications since it was proposed in 1952. However, Rohsenow’s model was based on analogy to forced convection and did not consider some main mechanisms, such as microlayer evaporation, transient conduction, etc., in nucleate pool boiling. Also, Rohsenow’s correlation does not explain the mechanism of how \( C_f \) is chosen while the value of \( C_f \) can greatly affect the predicted heat flux.

Surface contact angle is an important parameter to represent surface conditions as suggested by a number of researchers [7–9,25]. In the present work, water is chosen as the base fluid. The present model calculates different heat flux characteristics at different contact angles. The predicted results of the present analytical model were compared with the results calculated by Rohsenow’s correlation in Fig. 2. Researchers have reported different contact angles [7–9,14,15] in the range of 60°–80° when the surface is clean while in the presence of nanofluids the contact angle is in the range of 16°–45°. As can be seen in Fig. 2, the average heat flux when the contact angle equals to 60° is very close to the average heat flux calculated by the Rohsenow’s correlation when \( C_f \) equals to 0.0136. The heat flux at a contact angle of 70° corresponds to the heat flux when \( C_f \) equals to 0.013, and the results calculated with a \( C_f \) of 0.0126 are close to the predicted results based on the present model when the contact angle is 80°. The results show that the average heat flux decreases with a decrease in the contact angle at the same surface temperature. When the superheat is 15 K, the average heat flux decreases from 521 kW/m² when the contact angle is 80° to 408 kW/m² when the contact angle is 60°. The deterioration is up to 21.6%. However, the maximum heat flux increases with a decrease in the contact angle. The maximum heat flux reaches 690 kW/m² when the contact angle is 80°. When the contact angle decreases to 60°, the maximum heat flux increases to 917 kW/m² at a superheat of 18.84 K.

Fig. 2 shows some differences between our model and Rohsenow’s equation. The differences between our model and Rohsenow’s correlation are as follows:

1. Even though Rohsenow’s correlation is widely used in engineering applications, there is a need for a more robust result for the average heat flux. Furthermore, Rohsenow’s correlation applies for clean surfaces. When it is used to estimate the heat flux, it will not be accurate. Also, Rohsenow’s correlation is an empirical correlation, while our model is based on the pertinent physical mechanisms.
2. Our model establishes a relationship between the average heat flux and the surface contact angle, while Rohsenow’s correlation essentially shows the average heat flux changes for different surfaces with different \( C_f \). Furthermore, it does not explain how \( C_f \) is chosen and the correlation does not directly relate the average heat flux and the surface contact angle.
3. Our model was based on four heat transfer mechanisms that occur during different stages of nucleate boiling. Rohsenow’s correlation is mainly based on an analogy of nucleate boiling to forced convection, which may not accurately reflect the many complicated heat transfer mechanisms during the boiling process.
4. Rohsenow’s correlation is mainly based on \( q^* \propto T^3 \), while our model incorporates four different heat transfer mechanisms.

Based on a number of prior research works [7–9,14,15], the surface contact angle decreases after the introduction of a nanofluid instead of a regular fluid due to the deposition of nanoparticles in the boiling process as shown in Table 1. The present model is validated by comparing our results against a number of recent
The experimental data were selected from the works of Gerardi et al. [15] and of Bang and Chang [49]. Gerardi et al. [15] reported a contact angle of 80° for boiling with pure water and 16° for boiling with nanofluids. So 80° and 16° were used in our analytical model respectively. Bang and Chang [49] did not report contact angles, so the contact angle for the base fluid water in the present model is assumed to be 60° and the contact angle for nanofluid is 30° based on previous publications [7–9,14]. The water properties are used for the cases where the contact angle is 60° and 80°, and the properties of the nanofluid are used for the cases where the contact angle is 16° and 30°. The comparisons between our analytical model and the experimental results are shown in Fig. 3. As can be seen in Fig. 3, the analytical results are very close to the experimental results.

Fig. 3 shows that the average heat flux when the contact angle is 60° is very close to the experimental results using pure water in Bang and Chang [49]. The predicted heat flux at a contact angle of 30° is very close to the experimental results with nanofluid from Bang and Chang. The pure water heat fluxes from Gerardi et al. [15] are also close to the predicted results when the contact angle is 80° and the experimental results using nanofluid are close to analytical results when the contact angle is 16°. The change in nanofluid properties had little effect on the heat transfer. That is even though the thermal conductivity increases due to the addition of the nanoparticles, the decrease in the active nucleation sites is more dominant than the property change of the boiling liquid, so the average heat flux still decreases even as the thermal properties of the liquid enhances.

The heat transfer coefficient and the Nusselt number at different wall heat fluxes are plotted and compared with the experimental results from Suriyawong et al. [17] in Fig. 4. Again, since the cited reference did not provide contact angle values, it is assumed that 80° is the contact angle for a clean surface and 45° is the contact angle for the nanofluid boiling surface. The pure water experimental results agree well with the analytical results when the contact angle is 80°, and the experimental results utilizing 0.1% nanofluid agree well with the predicted analytical results when the contact angle is 45°.

The heat transfer coefficients and Nusselt number decrease with a decrease with contact angle. When the wall heat flux is 320 kW/m², the heat transfer coefficient decreases from $2.47 \times 10^4$ W/m² K at a contact angle of 80° to $2.14 \times 10^4$ W/m² K at a contact angle of 45°. The heat transfer coefficient deterioration is about 13.3% at a wall heat flux of 320 kW/m². The Nusselt number decreases from 47.2 to 45.1 at this wall heat flux. Although heat transfer deteriorates at the same wall heat flux, the maximum heat transfer coefficient and Nu at a smaller contact angle are higher than the heat transfer coefficient at a larger contact angle. Therefore, it appears that in nanofluid boiling, a larger maximum heat transfer coefficient and Nu can be obtained. The maximum heat transfer coefficient at a contact angle of 80° is $4.3 \times 10^4$ W/m² K, while the maximum heat transfer coefficient at a contact of 45° is $5.5 \times 10^4$ W/m² K. The Nusselt number is 94.7 and 151 respectively for these two contact angles. Our analytical results confirm these experimental trends.

### 4.2. CHF predictions

The critical heat flux can be enhanced in nanofluid boiling as shown by various researchers [8–15,18–20]. Many researchers suggest that when the contact angle decreases, the critical heat flux increases [7–9,14,15]. Experimental results for CHF from several researchers were compared with the CHF predicted by the present model. Different critical heat flux at different contact angles were plotted in Fig. 5 and compared with the experimental
results from Kim et al. [7] and Kim and Kim [50]. Kandlikar’s model [36] and Zuber’s correlation [34] were also plotted in Fig. 5. Fig. 5 shows that the critical heat flux increases from 691 kW/m² at a contact angle of 80°, which is measured for pure water on the heating surface, to 2450 kW/m² at a contact angle of 20°, which is the measured value for a contact angle after the nanofluid deposition [15]. The present model predicts a higher CHF enhancement than the Kandlikar’s model. The enhancement ratio is up to 3.54 if the CHF at a contact angle of 80° is chosen as the base value. Kandlikar’s model predicts an enhancement ratio of 1.95 when the contact angle decreases from 80° to 20°. Zuber’s model did not consider the surface conditions. The CHF predicted by Zuber’s correlation is almost the same with the CHF calculated with the present model at a 53° contact angle. The enhancement ratio predicted by the present analytical model is closer to the CHF enhancement reported in the literature [12,13,20,24]. The enhancement ratio variations of the present model with a change of contact angle were plotted in Fig. 6 which shows a continuous increase of CHF with a decrease in the contact angle.

The variations of CHF with departure diameters are plotted in Fig. 7. When the contact angle is lower, the surface superheat needed for the CHF occurrence is larger, and larger bubble diameters are needed. Larger bubble diameters and bubble departure frequencies at higher superheats result in higher heat fluxes.

5. Conclusions

In this work, a composite model is proposed to compute the average surface heat flux and the average heat flux variations with increasing superheats. The CHF was also predicted for different surface contact angles. Prior theoretical models on nucleate boiling are based on a single mechanism. However, four pertinent mechanisms affect the boiling process. Few researchers have systematically proposed a composite model analyzing these four main mechanisms. The main mechanisms in pool boiling were considered in the presented analytical model. Microlayer evaporation, transient conduction before and after bubble departs from the surface, microconvection induced by bubble growth and movement were all analyzed in the present model.

Our model has established a relationship between the average heat flux and surface contact angle. Our model has also extended the single bubble region to bubble interference region and enables quantification of the average heat flux at very high superheats. Transient conduction was found to be the main mechanism at larger superheats and natural convection accounts for a large percentage of heat transfer at very low superheats.

It was found that even though the thermal conductivity and other thermal properties were enhanced for nanofluids, the decreasing number of active nucleation sites at lower contact angles was more predominant and resulted in a decrease of the average surface heat fluxes for the same superheat. However, a larger heat transfer coefficient and Nu can be obtained for nanofluid boiling at a higher superheat. The results from our model were in good agreement with the available experimental results. Our model establishes a relationship between the average heat flux and surface contact angle leading to determination of CHF. It was also established that CHF enhancement ratio can be up to 3.54 when the surface contact angle decreases from 80° to 20°. This enhancement ratio agrees with a number of experimental results.

Conflict of interest

None declared.

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